## Subgraphs and Community Structure of Networks

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Social Computing course, CS60017

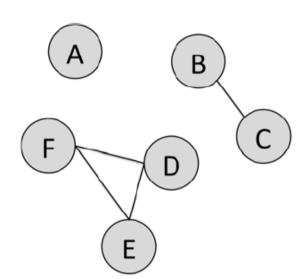
## Subgraphs

A subset of nodes and edges in a network

Given a (social) network, what are some subgraphs of interest?

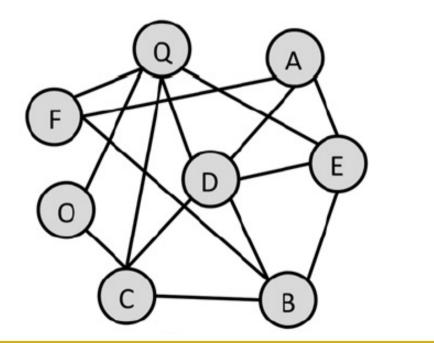
## Subgraphs

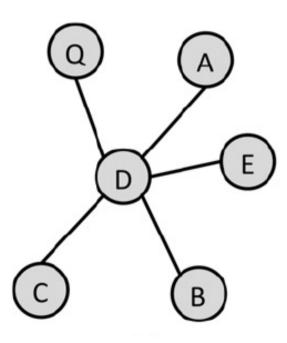
- A subset of nodes and edges in a network
- Given a (social) network, what are some subgraphs of interest?
  - Singletons: Isolated nodes
  - Connected components
  - Triads or triangles
  - Larger cliques



## Egocentric networks

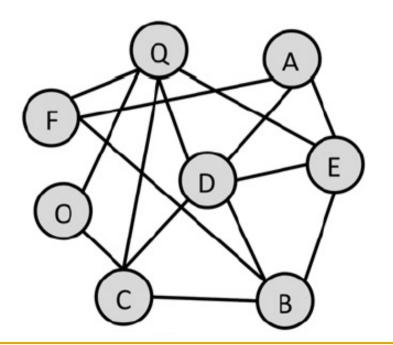
- From the perspective of a node (user)
- 1-degree egocentric network: a node and all its connections to its neighbors

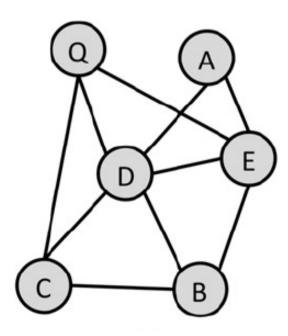




## Egocentric networks

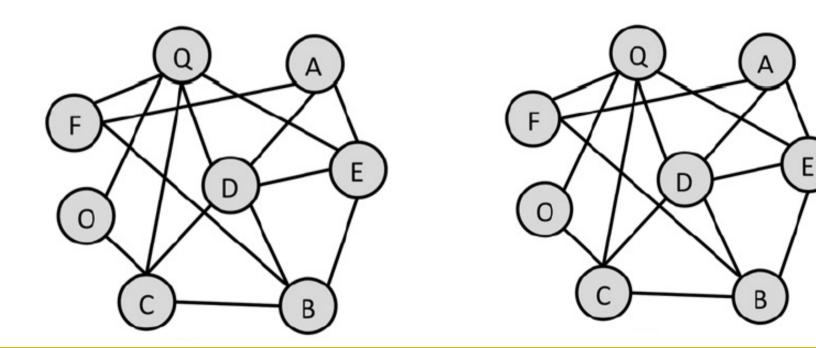
 1.5-degree egocentric network: a node, all its connections to its neighbors, and the connections among the neighbors





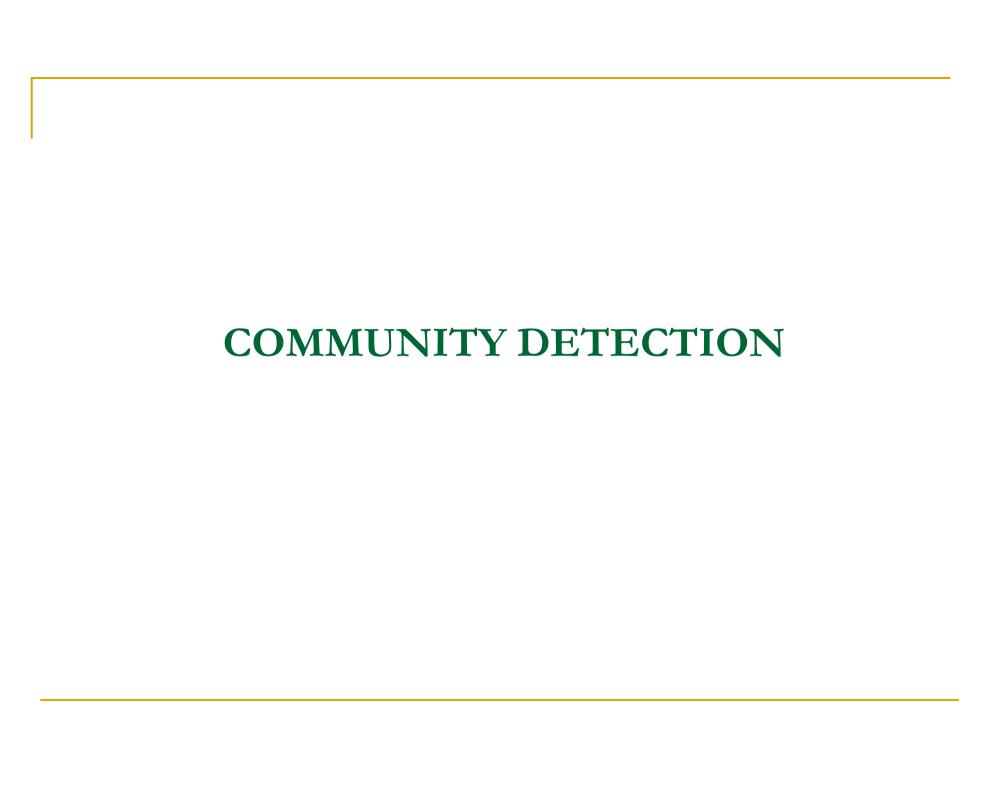
### Egocentric networks

 2-degree egocentric network: a node, all its neighbors, all neighbors of neighbors, and the connections among all these nodes



### Communities

- Community or network cluster
  - Typically a group of nodes having more and / or better interactions among its members, than between its members and the rest of the network
- No unique formal definition



## Community detection algorithms

 Lot of applications – identifying similar nodes, close friends, recommendation, ...

- Challenging
  - Communities are not well-defined
  - Number of communities in a network is not known

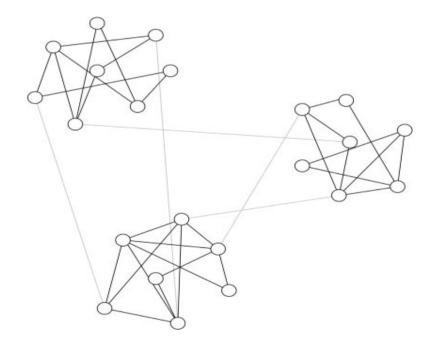
## Two broad types of algorithms

- Detection of disjoint communities
  - Each community is a partition of the network

- Detection of overlapping communities
  - A node can be members of multiple communities

## Algorithm by Girvan & Newman

- Community structure in social and biological networks, PNAS, 2002
- Focus on edges that are most "between" communities



### Edge betweenness

- Edge betweenness of an edge e: fraction of shortest paths between all pairs of vertices, which run through e
- Edges between communities are likely to have high betweenness centrality
- Progressively remove edges having high betweenness centrality, to separate communities from one another

## Girvan-Newman algorithm

- Compute betweenness centrality for all edges
- 2. Remove the edge with highest betweenness centrality
- 3. Re-compute betweenness centrality for all edges affected by the removal
- 4. Repeat steps 2 and 3 until no edges remain
- Time complexity
  - Graph of *n* vertices and *m* edges: betweenness centrality of all edges can be computed in *O(mn)* time
  - □ Hence, worst case time complexity:  $O(m^2n)$

### How many communities?

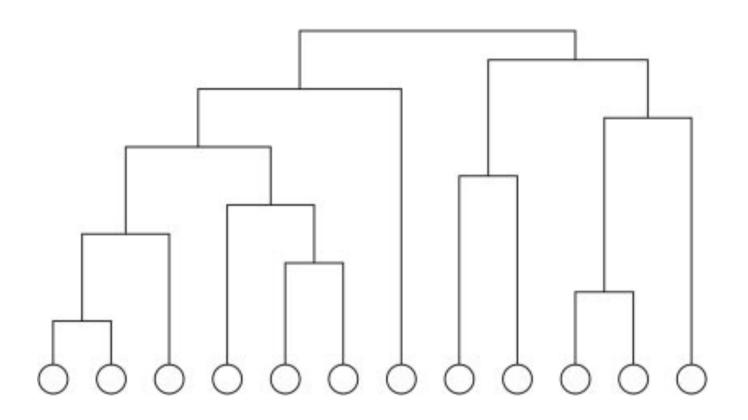
 Community structure of a graph is hierarchical, with smaller communities nested within larger ones

 Represented as a hierarchical clustering tree: dendrogram

 A "slice" through the tree at any level gives a certain number of communities

Which level to slice at?

## An example dendrogram



## Hierarchical clustering algorithms

- Agglomerative algorithms (bottom-up)
  - Clusters / communities iteratively merged if their similarity is sufficiently high
- Divisive algorithms (top-down)
  - Clusters / communities iteratively split by removing edges
- Both can be represented by dendrograms
- Need some way to decide at what level to slice the dendrogram – what is a good community structure?

### Objective functions for CD

- Community or network cluster
  - Typically a group of nodes having more and / or better interactions among its members, than between its members and the rest of the network
- Typical CD algorithms
  - Choose an objective function that captures the above intuition
  - Optimize the objective function using heuristics or approximation algorithms

# OBJECTIVE FUNCTIONS FOR COMMUNITY DETECTION

Empirical Comparison of Algorithms for Network Community Detection, Leskovec et al., WWW 2010

### Various objective functions

- Two criteria of interest for measuring how well a particular set S of nodes represents a community
  - Number of edges among the nodes within S
  - Number of edges between nodes in S and rest of network
- Two types of objective functions
  - Single criterion considers any one of the above criteria
  - Multi criterion considers both the above criteria

#### Multi-criterion scores

 Consider both the criteria for measuring quality of a set S of nodes

 Lower values of f(S) signify a more community-like set of nodes

#### **Notations**

- G = (V, E) is the network.
- n = |V| = number of nodes
- = m = |E| = number of edges
- $d(u) = k_u =$ degree of node u
- S: set of nodes
- $n_s = number of nodes in S$
- $m_s$  = number of edges within S (both nodes in S)
- $c_s$  = number of edges on the boundary of S

## Expansion

$$f(S) = \frac{c_S}{n_S}$$

 Number of edges per node in S, that points outside the set S

## Internal density

$$f(S) = 1 - \frac{m_S}{n_S(n_S-1)/2}$$

Internal edge density of the set S

## **Cut Ratio**

$$f(S) = \frac{c_S}{n_S(n - n_S)}$$

Fraction of all possible edges leaving the set S

### Conductance

$$f(S) = \frac{c_S}{2m_S + c_S}$$

- Fraction of total edge volume that points outside the cluster
- Edge volume = sum of node-degrees
- Effectively, tatio between number of edges inside S and the number of edges leaving S

## Normalized Cut

$$f(S) = \frac{c_S}{2m_S + c_S} + \frac{c_S}{2(m - m_S) + c_S}$$

What does this measure physically signify?

## Maximum Out Degree Fraction (ODF)

$$\max_{u \in S} \frac{|\{(u,v): v \notin S\}|}{d(u)}$$

 Maximum fraction of edges of a node in S, that points outside the set S

## Average ODF

$$f(S) = \frac{1}{n_S} \sum_{u \in S} \frac{|\{(u,v): v \notin S\}|}{d(u)}$$

 Average fraction of edges of nodes in S, that points outside S

#### Flake ODF

$$f(S) = \frac{|\{u:u \in S, |\{(u,v):v \in S\}| < d(u)/2\}|}{n_S}$$

 Fraction of nodes in S that have fewer edges pointing inside S, than to outside S

### Observations by Leskovec et al.

- Internal density and Maximum-ODF are not good measures for community quality
  - Does not show much variation, except for very small communities
- Cut ratio has high variance
  - communities of similar sizes can have very different numbers of edges pointing outside
- Flake-ODF prefers larger communities
- Conductance, expansion, normalized cut, average-ODF all exhibit qualitatively similar behavior

## Single-criterion scores

- Consider only one of the two criteria for measuring quality of a set S of nodes
- Two simple single-criterion scores:
  - Volume: Sum of degrees of the nodes in S
  - Edges Cut: c<sub>s</sub>: Number of edges needed to be removed to disconnect nodes in S from the rest of the network

## Modularity-based measures

 A set of nodes is a good community if the number of edges within the set is significantly more than what can be expected by random chance

• Modularity Q = 
$$\frac{1}{4m}(m_S - E(m_S))$$

- Number of edges within set S, minus expected number of edges within the set S
- The 1/4m factor is merely conventional

## Modularity ratio

$$\frac{m_S}{E(m_S)}$$

- Alternative measure of how well set S represents a community
- Ratio of the number of edges among nodes in S, and expected number of such edges in null model

## Expected number of edges

- Null model: Erdos-Renyi random network having the same node degree sequence as the given network
- Realised in practice using Configuration Model
- Expected to have no community structure

## Mathematical definition of Modularity

- For two particular nodes i and j:
  - $\square$  Number of edges between the nodes:  $A_{ij}$
  - □ Degrees: *k<sub>i</sub>, k<sub>i</sub>*
  - □ Expected number of links between i and j:  $k_i k_j /2m$
- Do the nodes i and j have more edges than expected by random chance?

$$A_{ij} - k_i k_j /2m$$

## Modularity for a given network

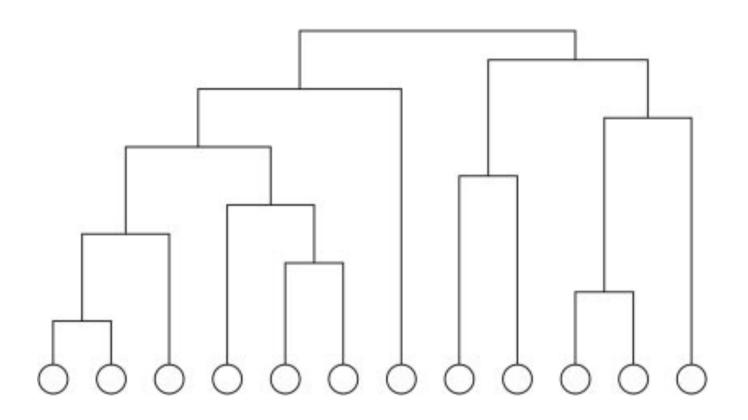
$$Q = \frac{1}{2m} \sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) \delta(C_i, C_j)$$

■ The delta function is 1 if both nodes i and j are in the same community ( $C_i = C_j$ ), 0 otherwise

### Using modularity for CD

- Modularity can be used to decide at which level to slice the dendrogram
- Optimize modularity
  - Exhaustive maximization is NP-hard
  - Heuristics and approximations used

### An example dendrogram



### Greedy algorithm for maximizing Q

- Fast algorithm for detecting community structure in networks, Newman, PRE 69(6), 2004
- Greedy agglomerative hierarchical clustering
  - Start with n clusters, each containing a single node
  - Add edges such that the new partitioning gives the maximum increase (minimum decrease) of modularity wrt the previous partitioning
  - A total of n partitionings found, with number of clusters varying from n to 1
  - Select the partitioning having highest modularity

### Most popular Q optimization algorithm

#### Louvain algorithm:

https://perso.uclouvain.be/vincent.blondel/research/louvain.html

#### Optimization in two steps

- Step 1: look for small communities optimizing Q locally
- Step 2: aggregate nodes in the same community and build a new network whose nodes are the communities
- Repeat iteratively until a maximum of modularity is attained and a hierarchy of communities is produced
- Time: approx O(n log n)

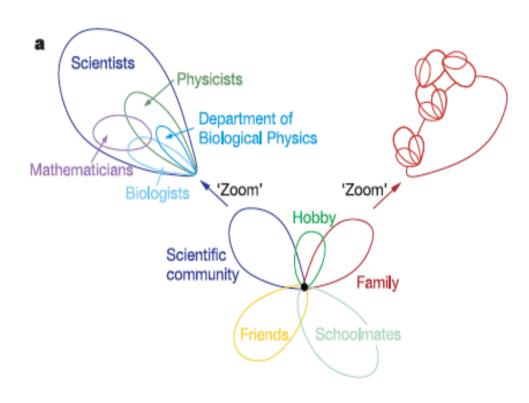
### For reading

- Many subsequent works have suggested improvements for maximizing modularity
  - Reducing time complexity
  - Normalizing with number of edges to minimize bias towards larger communities
  - ...
- Read "Community detection in graphs" by Fortunato, Physics Reports, 2010.

# OVERLAPPING COMMUNITY DETECTION

### Overlapping communities

 Nodes in real networks are often parts of multiple overlapping communities



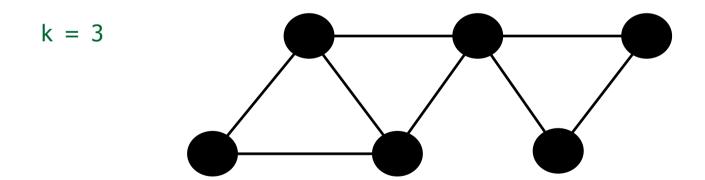
### Two algorithms

- Clique Percolation Method
  - Uncovering the overlapping community structure of complex networks in nature and society, Palla et al., Nature Letters, vol. 435, 2005
- Link communities
  - Link communities reveal multiscale complexity in networks, Ahn et al., Nature Letters, vol. 466, 2010

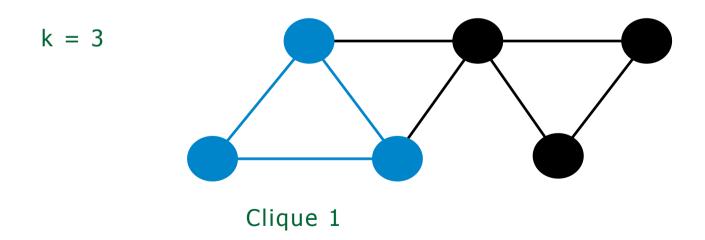
### Clique Percolation Method

- Concept:
  - Internal edges of communities likely to be part of cliques
  - Inter-community edges unlikely to be part of cliques
- Adjacent k-cliques: two k-cliques are adjacent if they share k-1 nodes

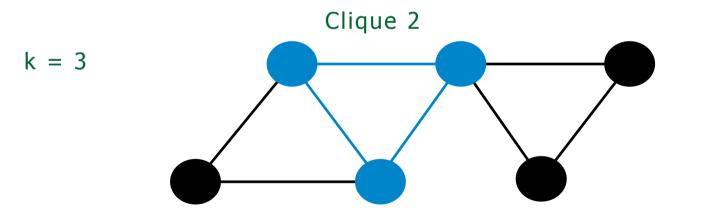
#### Adjacent k-cliques



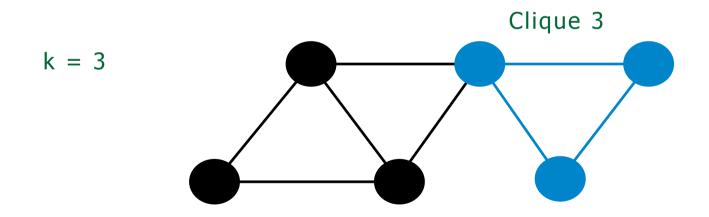
#### Adjacent k-cliques



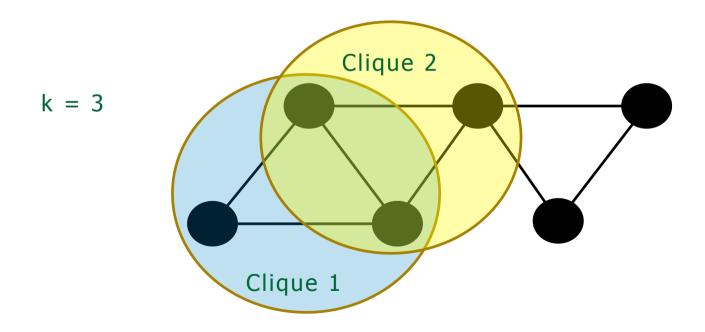
#### Adjacent k-cliques



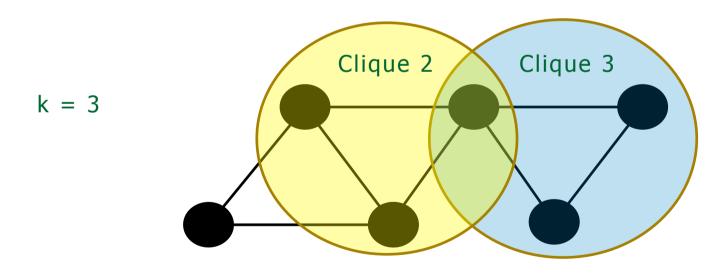
#### Adjacent k-cliques



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#### Adjacent k-cliques

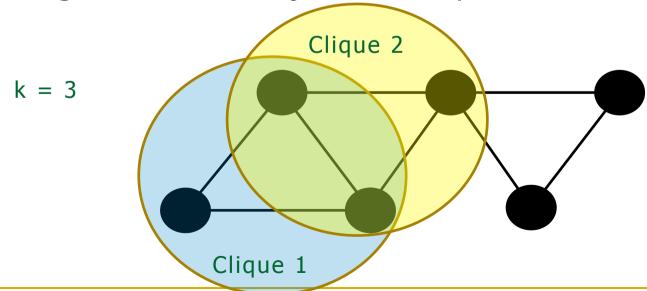


#### k-clique community

Union of all k-cliques that can be reached from each other

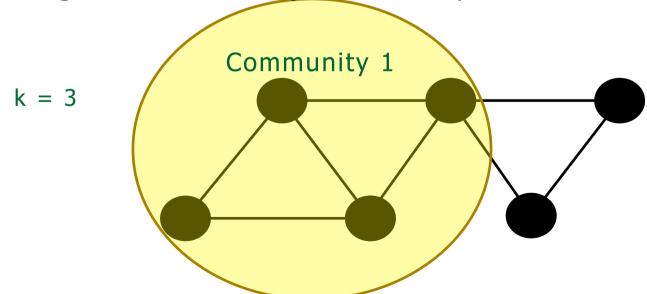
#### k-clique community

Union of all k-cliques that can be reached from each other



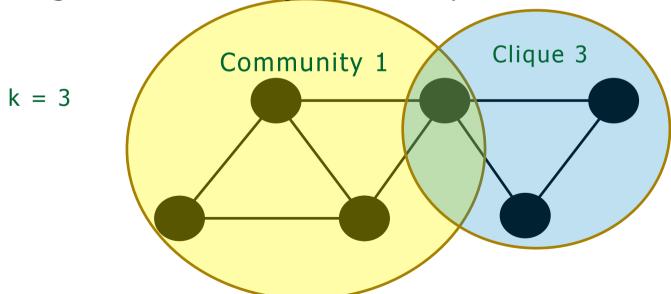
#### k-clique community

Union of all k-cliques that can be reached from each other



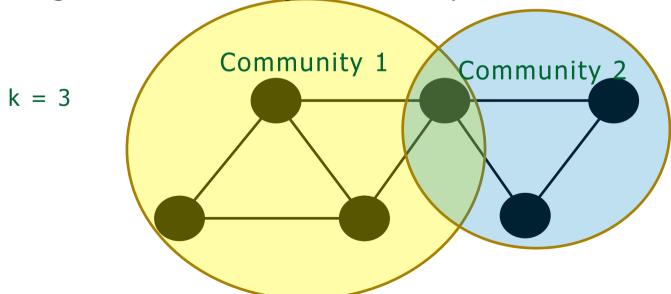
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Union of all k-cliques that can be reached from each other



# Algorithm

- Locate maximal cliques
- Convert from cliques to k-clique communities

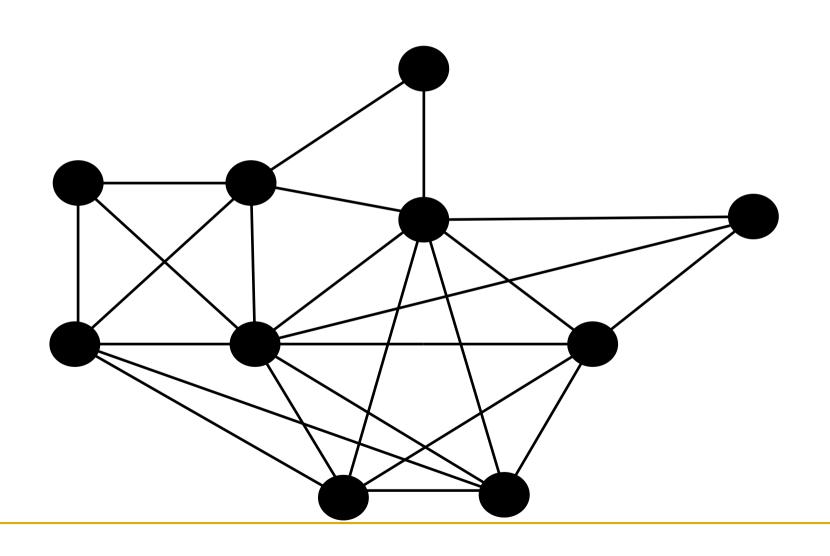
## Locate Maximal Cliques

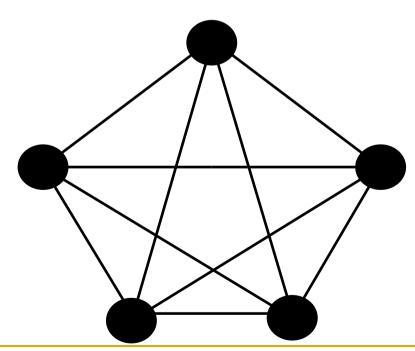
 Largest possible clique size can be determined from degrees of vertices

 Starting from this size, find all cliques, then reduce size by 1 and repeat

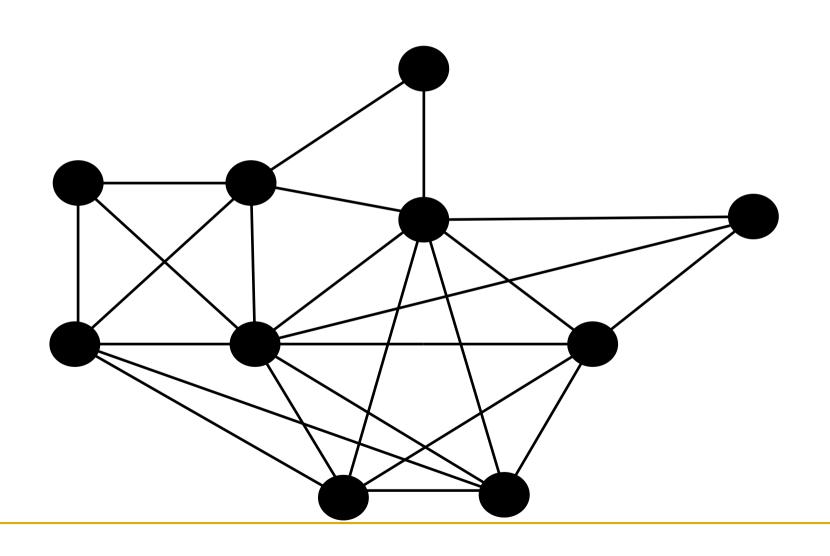
# Algorithm

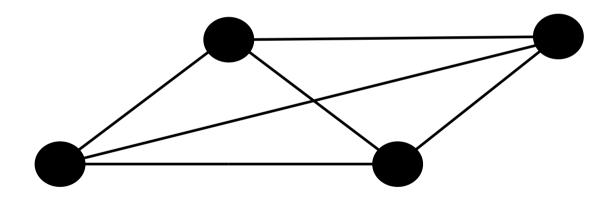
- Locate maximal cliques
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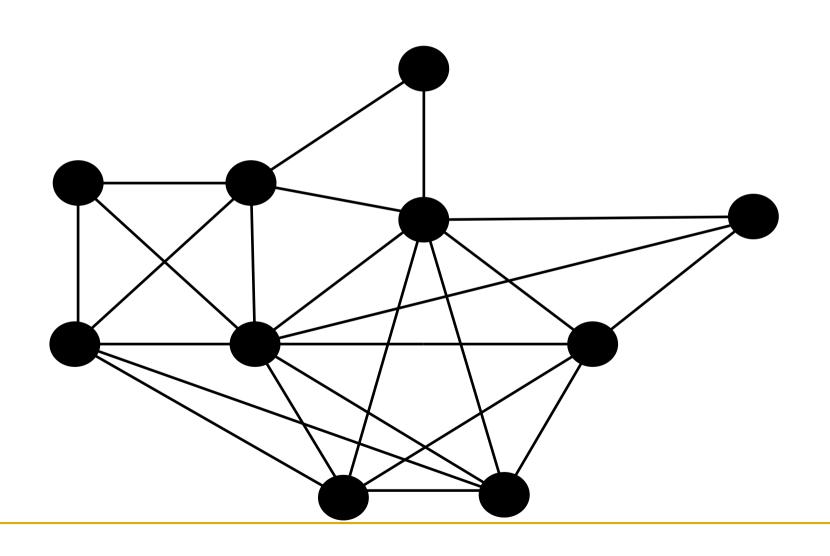


Clique 1: 5-clique

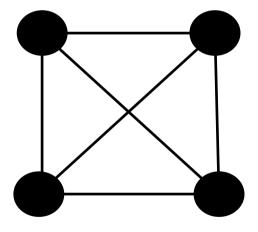


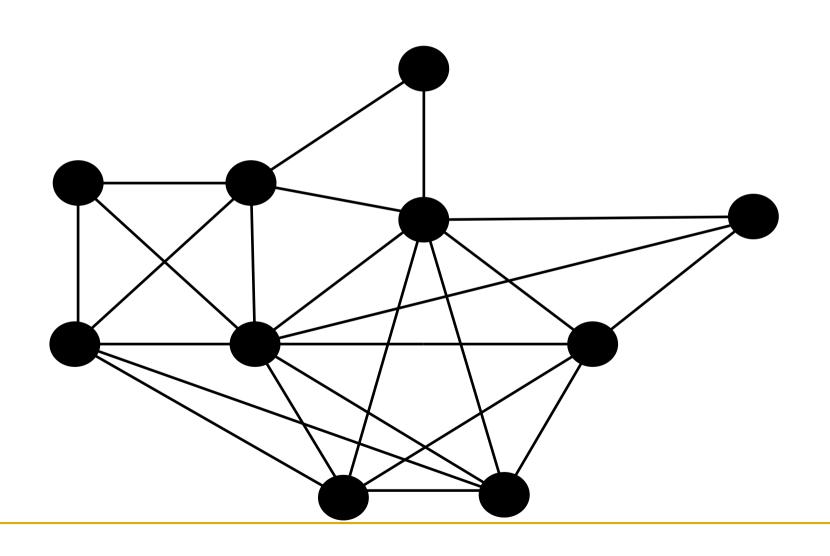


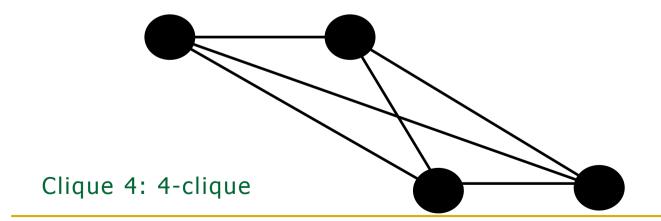
Clique 2: 4-clique

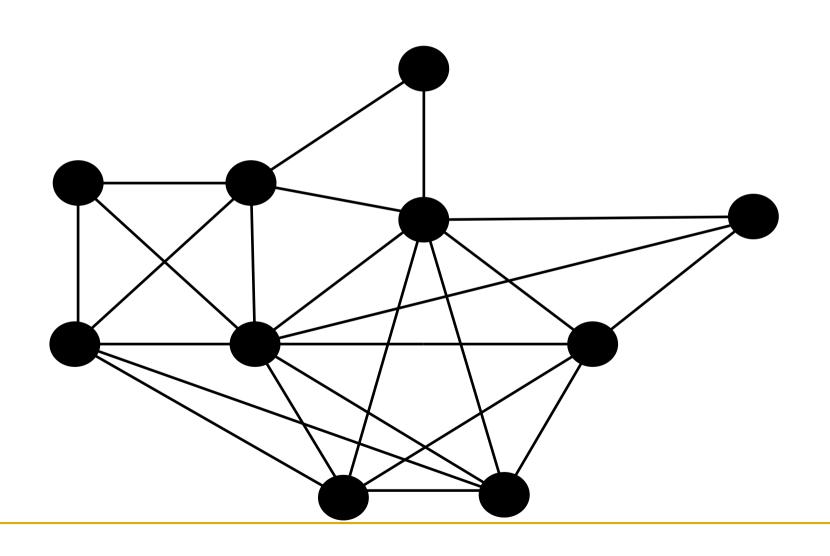


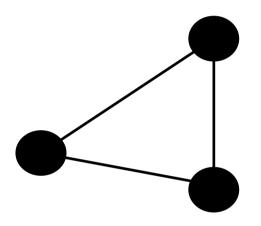
Clique 3: 4-clique



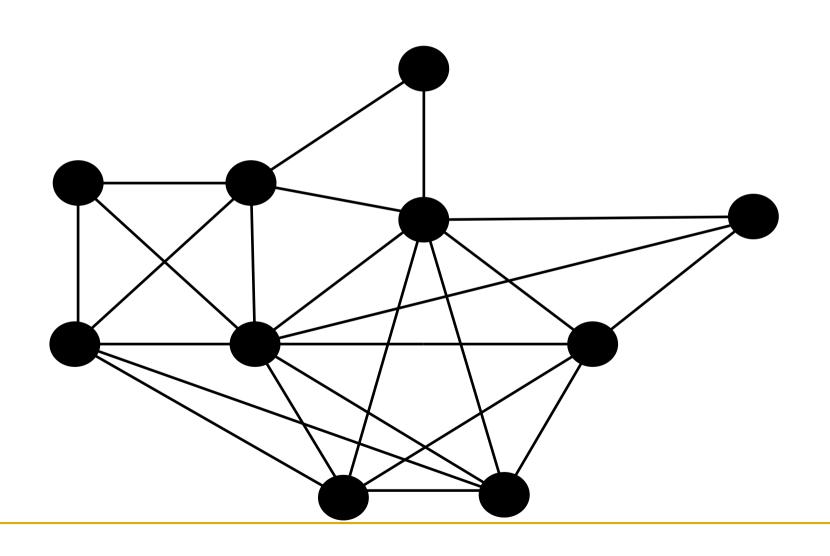




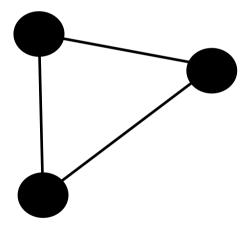




Clique 5: 3-clique



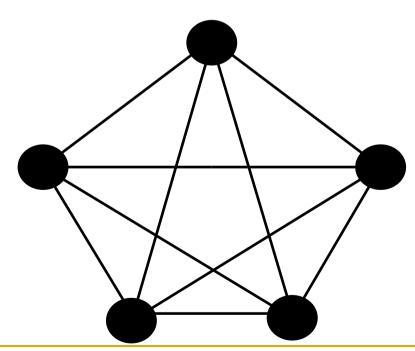
Clique 6: 3-clique



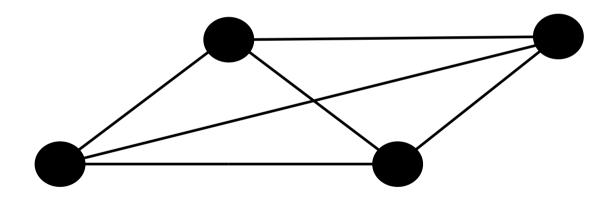
	1	2	3	4	5	6
1	5					
2		4				
3			4			
4				4		
5					3	
6						3

#### Clique-Clique overlap matrix

	1	2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	3	2
6	2	2	2	1	2	3



Clique 1: 5-clique



Clique 2: 4-clique

#### Clique-Clique overlap matrix

	1	2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	3	2
6	2	2	2	1	2	3

- For a given value of k, k-clique communities:
  - Connected clique components in which neighboring cliques are linked to each other by at least k-1 common nodes
- How to find k-clique communities from the cliqueclique overlap matrix?
  - Erase every diagonal element smaller than k
  - Erase every off-diagonal element smaller than k-1
  - Replace remaining elements by 1
  - Carry out a component analysis of this matrix

	1	2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	3	2
6	2	2	2	1	2	3

		2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	3	2
6	2	2	2	1	2	3

k=4

		2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	0	2
6	2	2	2	1	2	0

Delete if less than k

	1	2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	0	2
6	2	2	2	1	2	0

	1	2	3	4	5	6
1	5	3	1	3	1	2
2	3	4	1	1	1	2
3	1	1	4	2	1	2
4	3	1	2	4	0	1
5	1	1	1	0	0	2
6	2	2	2	1	2	0

k=4

	1	2	3	4	5	6
1	5	3	0	3	0	0
2	3	4	0	0	0	0
3	0	0	4	0	0	0
4	3	0	0	4	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Delete if less than k-1

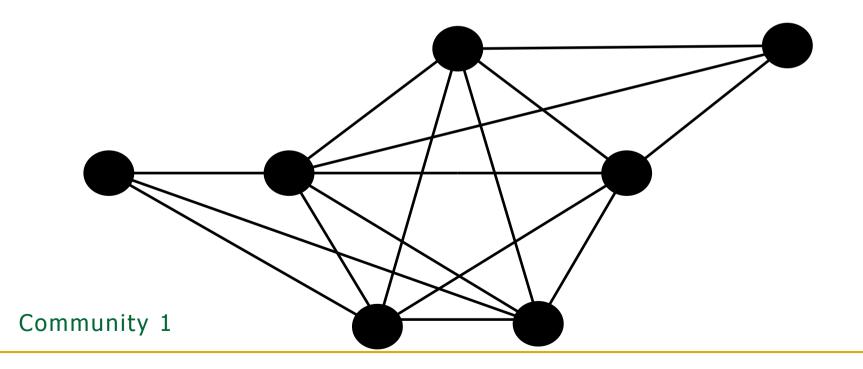
	1	2	3	4	5	6
1	5	3	0	3	0	0
2	3	4	0	0	0	0
3	0	0	4	0	0	0
4	3	0	0	4	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

k=4

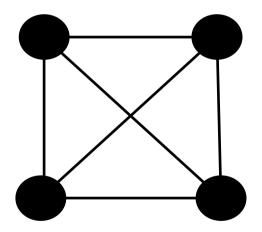
	1	2	3	4	5	6
1	1	1	0	1	0	0
2	1	1	0	0	0	0
3	0	0	1	0	0	0
4	1	0	0	1	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Change all non-zeros to 1

	1	2	3	4	5	6
1	1	1	0	1	0	0
2	1	1	0	0	0	0
3	0	0	1	0	0	0
4	1	0	0	1	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0



k=4



Community 2

## Clique Percolation Method: Analysis

- Believed to be non-polynomial
- No closed formula can be given
- However, claimed to be efficient on real systems

#### Limitations

- Fail to give meaningful covers for graph with few cliques
- With too many cliques, might give a trivial community structure

#### Link communities

- A node might belong to multiple communities
  - For a person: family, co-workers, friends, ...
- A link often exists for one dominant reason
  - Two people are in the same family, or are co-workers
- Link community: a set of closely inter-related links

#### Identifying Link communities

- Hierarchical clustering with a similarity between links to build a dendrogram
  - Each leaf of the dendrogram is a link from the original network
  - Branches of the dendrogram are link communities
- Slice the dendrogram at a suitable level
- Each link placed in a single community
- Each node inherits membership of the communities of all its links

#### Similarity measure between links

- Node *i* and its neighboring nodes:  $n_+(i)$
- Similarity measured only between pairs of links which share a node
- Similarity between  $e_{ik}$  and  $e_{jk}$ :

$$S(e_{ik},e_{jk}) = |n_+(i) \cap n_+(j)|/|n_+(i) \cup n_+(j)|$$

## Which level to slice the dendrogram?

- Measure: Partition density D
  - Total number of links in network: m
  - $P_1, P_2, ..., P_C$ : partition of links into C subsets
  - $P_c$  has  $n_c$  nodes and  $m_c$  links

$$D_c = \frac{m_c - (n_c - 1)}{n_c(n_c - 1)/2 - (n_c - 1)}$$

□ Partition density is average of  $D_c$  weighted by the fraction of links in  $P_c$ 

$$D = \frac{2}{M} \sum_{c} m_{c} \frac{m_{c} - (n_{c} - 1)}{(n_{c} - 2)(n_{c} - 1)}$$

# DIFFERENT TYPES OF GROUPS IN A SOCIAL NETWORK

## Different methods to identify groups

- Identifying groups based on network structure community detection algorithms
- How about identifying groups based on content, e.g., text or profile attributes?
- Deep Twitter Diving: Exploring Topical Groups in Microblogs at Scale, Bhattacharya et al., CSCW 2014

### Identified topical groups in Twitter

Topical Groups = Experts + Seekers

Experts: Users who have expertise on the topic

Seekers: Users who are interested in the topic



@BarackObama
Expert on Politics

@BarackObama
Seeker on Basketball



# Identifying topical groups at scale

Crawled data for first 38 million users in Twitter

88 Million lists, 1.5 Billion social links

Identified 36 thousand topical groups

# Diversity: Topics and Group Size

No. of			Number of expert	ts		
seekers	< 100	100 - 500	500 – 1K	1K – 5K	5K – 10K	>10K
< 1K	(5416) geology, karate, malaria, neurology, tsunami, psychiatry, radiology, pediatrics, dermatology, dentistry	(132) volleyball, philosophers, tarot, perfume, florists, copywriters, taxi, esperanto				
1K – 5K	(915) biology, chemistry, swimmers, astrophysics, multimedia, semiconductor, renewable-energy, breast-cancer, judaism	(428) painters, astrology, sociology, geography, forensics, anthropology, genealogy, archaeology, gluten, diabetes, neuroscience	(17) architects, insurance, second-life, police, progressives, creativity			
5K – 10K	(166) malware, gnu, robot, chicago-sports, gospel-music, space- exploration, wall-street	(202) horror, agriculture, atheism, attorneys, furniture, art-galleries, ubuntu	(34) psychology, poetry, catholic, hospitals, autism, jazz	(2) coffee, dealers		
10K - 50K	(174) ipod, ipad, virus, Liverpool-FC, choreographers, heavymetal, backstreet-boys, world-cup,	(312) olympics, physics, theology, earthquake, opera, makeup, Adobe, wrestlers, typography, american-idol	(146) tennis, linux, astronomy, yoga, animation, manga, doctors, realtors, wildlife, rugby, forex, php, java,	(67) law, history, beer, golf, librari- ans, theatre, military, poker, conservatives, vegan		
50K- 100K	(7) bbc-radio, UK- celebs, christian- leaders, superstars	(61) hackers, programmers, bicycle, GOP, fantasy-football, NCAA, wwe, sci-fi	(35) medicine, cyclists, investors, recipes, NHL, xbox, triathlon, Google	(37) hotels, museums, hockey, architecture, charities, weather, space		
> 100K	(3) headlines, brits	(49) pop-culture, gospel, BBC, reality-tv, bollywood	(58) religion, actresses, gadgets, graphic-design, directors, lifestyle, gossip, commentators, youtube	(140) books, govern- ment, comedy, en- vironment, baseball, soccer, hollywood, iphone, economics, money	(25) fashion, education, wine, photography, radio, restaurants, science, SEO	(17) music, tech, business, politics, food, sports, celebs, health, media, bloggers, travel, writers

#### A Small Number of Very Popular Groups

No. of	Number of experts								
seekers	< 100	100 – 500	500 – 1K	1K – 5K	5K – 10K	> 10K			
< 1K	(5416) geology, karate, malaria, neurology, tsunami, psychiatry, radiology pediatrics	(132) volleyball, philosophers, tarot, perfume, florists, copywriters, taxi esperanto							
	dermate (37) h	otels, mu-							
1K – 5K	istry, seums,	hockey,							
	media, architec	´ I							
	oroust c	ather, space							
5K –	$\begin{array}{ c c c c c }\hline (166) & (140) bc \\\hline (166) & (140) bc \\\hline \end{array}$	ooks, govern-	<b>(25</b> ) <i>fashio</i>	$n, \mid (17) m$	usic, tech,				
	gospel- explora ment, comedy, en-		education,	busines	business, politics,				
10K -	(174) <i>vironme</i>	(174) vironment, baseball,		g- food,	sports,				
50K	virus, choreog Soccer,	hollywood,	raphy, radi	io, <i>celebs</i> ,	health,				
	metal, iphone,	economics,	restaurants,	media,	bloggers,				
50K- 100K	(7) bl money		science, SEO	travel, v	writers				
	leaders, superstars	GOP, fantasy-football, NCAA, wwe, sci-fi	xbox, triathlon, Google	architecture, charities, weather, space					
> 100K	(3) headlines, brits	(49) pop-culture, gospel, BBC, reality-tv, bollywood	(58) religion, actresses, gadgets, graphic-design, directors, lifestyle, gossip, commentators, youtube	(140) books, govern- ment, comedy, en- vironment, baseball, soccer, hollywood, iphone, economics, money	(25) fashion, education, wine, photog- raphy, radio, restaurants, science, SEO	(17) music, tech, business, politics, food, sports, celebs, health, media, bloggers, travel, writers			

#### Thousands of Specialized Niche Groups

No. of	Number of experts									
seekers	< 100	100 – 500	500 – 1K		1K – 5K	5K – 10K	>10K			
< 1K	(5416) geology, karate,	(132) volleyball, philosophers, tarot,								
	malaria, neurology,									
	radiology, per (54	16) goology k	6) goology karata		volley	zhall				
	dermatology, dei	ov dei			-	van,				
1K -	(915) <i>biology</i> , <i>ma</i>	ology,	philosophers, tarot,							
5K	interes musi	<i>nami</i> , psychiatry, liology, pediatrics,		perfume, florists, copy- writers, taxi, esperanto						
	media, semicor rad									
	Tellewable-ellerg,			,,						
5K -	breast-cancer, ju dermatology, dentistry									
10K	robot, chicago (91	5) biology, chem-		(428) painters, astrol-						
	gospei-music,									
1077	exploration, wall	istry, swimmers,			ogy, sociology, geogra-					
10K - 50K	(174) ipod, virus, Liverp ast	ipod, Liverpo astrophysics, multi-			phy, forensics, anthro-					
3010	-11									
	metal, backstree me	dia, semicond	pology, genealogy, ar-							
	world-cup, ren				chaeology, gluten, dia-					
50K-	(7) DDC-radio,	3-radio,								
100K	celebs, ch	ast-cancer, juda	neuroscience							
	leaders, supersta	NCAA, wwe, sci-fi	720072, 4224442	<del></del>	ties, weather, space					
>	(3) headlines, brits	(49) pop-culture,	(58) religion	n, actresses,	(140) books, govern-	(25) fashion,	(17) music, tech,			
100K		gospel, BBC, reality-tv,		graphic-	ment, comedy, en-	education,	business, politics,			
		bollywood	design,	directors,	vironment, baseball,	wine, photog- raphy, radio,	food, sports,			
					ossip, com- soccer, hollywood,		celebs, health,			
		mentators,				restaurants, science, SEO	media, bloggers, travel, writers			
					inoney	science, sho	uavel, witters			

## Breaking the Twitter stereotype

- Twitter stereotype
  - Popular news on few topics such as sports, entertainment, politics, technology
  - Celebrity gossip, current news, and chatter

- Breaking the stereotype
  - Majority of the population discuss few popular topics, but
  - Smaller groups interested in thousands of niche, specialized topics

## Detecting topical groups

 We followed content-based approach to identify topical groups

Could community detection algorithms be used on the social network to detect them?

 Applied BGLL / Louvain algorithm on the Twitter subscription network to identify communities

### Detecting topical groups

- Louvain largely unable to detect topical groups, especially the smaller ones (on niche topics)
- Communities detected by Louvain fare better on structural measures like cut-ratio, conductance
- Topical groups do not have good structural quality
  - Poor values for standard community quality metrics such as cut-ratio and conductance

## Why do groups form?

- "Common Identity and Bond Theory"
  - Prentice et. al. "Asymmetries in Attachments to Groups and to Their Members: Distinguishing Between Common-Identity and Common-Bond Groups", Personality and Social Psychology Bulletin, 1994
- Identity based groups
- Bond based groups

#### Common Identity and Bond Theory

#### **Identity Based Groups**

Low Reciprocity
Low Personal Interactions
High Topicality of discussions

Examples:
Fans at a football match,
Attendees at a conference

#### **Bond Based Groups**

High Reciprocity
High Personal Interactions
Low Topicality of discussions

Examples: Family, personal friends

# Analysis of 50 topical groups

- Low reciprocity among members
- Few one-to-one interactions
- Most tweets posted by experts are related to topic
- → Topical groups are identity-based which are difficult to detect via community detection algorithms